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HEATING OF A MICROPOLAR LIQUID DUE TO VISCOUS ENERGY DISSIPATION IN CHANNELS. 1. POISEUILLE FLOW

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An analytical study is made of the effect of the internal microstructure of a liquid on its heating due to viscous energy dissipation.

In its different forms, the theory of micropolar liquids (MPL) [1] is currently used for theoretically describing transfer processes in liquids with an internal microstructure: liquid crystals, magnetic liquids, certain suspensions, and associated liquids. However, a comparatively large number of transfer coefficients (material constants) which until recently had no method of being determined are included in this theory. The studies [2, 3] proposed methods of determining different parameters characterizing the internal microstructure of a liquid. For example, the method of determining the material constants of a liquid in [3] is based on measurement of heating of the liquid as a result of viscous energy dissipation during Poiseuille flow in a plane channel in the case of constant channel-wall temperature.

The present work analytically solves a problem of the heating of an MPL flowing as a result of a fixed pressure gradient in the plane channel. We set thermal boundary conditions more general than those in [3] and take into account the change in temperature through the thickness of the channel walls. A numerical analysis is made of the dependence of the temperature field in the liquid on quantities characterizing its micropolarity. The magnitude of the dissipative heating of a liquid flowing in microcapillaries ($h \leq 10^{-5}$ m) is very small in the overwhelming majority of cases. However, we also have the goal of studying dissipative heating of the liquid under conditions where its value is sufficiently large for experimental determination (for example, with a pressure drop Δp = 20-40 atm).

In the second article we will solve a similar problem for Couette flow with a prescribed constant relative velocity of the channel walls. The role of microrotations of particles of the medium in the case of significant dissipative heating of the microstructural liquid is established for a broad range of practical instances, such as when two surfaces with an intervening liquid are moving at a comparatively high velocity relative to each other.

Let us examine the stabilized flow of an incompressible micropolar liquid under the influence of a constant pressure gradient dp/dx between parallel plates located a distance 2h from one another. The x axis of the Cartesian coordinate system coincides with the central line of the channel, while the y axis is perpendicular to the plates. In this case, the velocity vector $\vec{v}$ and the microrotation vector $\vec{\nu}$ only have nontrivial components $v_x(y)$ and $\nu_z(y)$, respectively. We will assume that the physical properties of the MPL are constant, i.e., we will ignore the effect of dissipative heating on them, as well as the body forces and their moments. Given these assumptions, the equations describing the flow of the MPL have the form

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$$(\mu+\varkappa)\frac{d^2v_x}{dy^2}+\varkappa\frac{d\nu_z}{dy}-\frac{dp}{dx}=0, \qquad (1)$$

$$\gamma\frac{d^2\nu_z}{dy^2}-\varkappa\frac{dv_x}{dy}-2\varkappa\nu_z=0. \qquad (2)$$

We use boundary conditions in the form

$$\vec{v}|_g=\vec{V},\ \vec{\nu}|_g=\frac{\alpha}{2}\,(\mathrm{rot}\,\vec{v})|_g+\vec{\Omega}\,(1-\alpha). \qquad (3)$$

Since the linear $\vec{V}$ and angular $\vec{\Omega}$ velocities of the boundary, as well as all of the components of the vectors $\vec{v}$ and $\vec{\nu}$ except for $v_x(y)$ and $\nu_z(y)$ are equal to zero, boundary conditions (3) can be written as

$$v_x(\pm h)=0,\ \nu_z(\pm h)=-\frac{\alpha}{2}\left(\frac{dv_x}{dy}\right)_{y=\pm h}. \qquad (4)$$

Having solved system of differential equations (1), (2) with boundary conditions (4), we obtain expressions for $v_x(y)$ and $\nu_z(y)$:

$$\frac{v_x}{v_0}=1-\tilde{y}^2+\frac{2\delta_0\xi}{2+\delta_0\xi}\,\frac{\mathrm{ch}\,k\tilde{y}-\mathrm{ch}\,k}{k\,\mathrm{sh}\,k}, \qquad (5)$$

$$\frac{\nu_z h}{v_0}=\tilde{y}-\frac{\xi(2+\delta_0)}{2+\delta_0\xi}\,\frac{\mathrm{sh}\,k\tilde{y}}{\mathrm{sh}\,k}, \qquad (6)$$

where $v_0=\frac{(-dp/dx)h^2}{2\mu^n};\ \mu^n=\mu+\frac{\varkappa}{2};\ \delta_0=\frac{\varkappa}{\mu^n};\ k=\tilde{k}\,h=\left(\frac{2\mu+\varkappa}{\mu+\varkappa}\,\frac{\varkappa}{\gamma}\right)^{1/2}h;\ \tilde{y}=y/h;\ \xi=1-\alpha.$

It is apparent from (5) and (6) that the velocity and microrotation profiles depend on δ_0, ξ, k, and h. Figure 1 shows curves of v_x/v_0 and $\nu_z h/v_0$ constructed from Eqs. (5) and (6) with constant δ_0 and $\tilde{k}$ for different ξ and h. The change in h corresponds to different k with a constant $\tilde{k}$. It follows from the figures and numerical analysis of Eqs. (5) and (6) that the liquid exhibits micropolarity to a greater extent with a decrease in h and an increase in ξ. The microrotation $\vec{\nu}$ can coincide with the curl $\vec{w}$ on the boundary (with $\xi = 0$) only in the absence of moment stresses between liquid particles, i.e., when the liquid is Newtonian. In fact, as can be seen from (5) and (6), as $\xi \to 0$ the linear velocity v_x coincides with the velocity of a Newtonian liquid with a shear viscosity $\mu^n = \mu + \kappa/2$, while the microrotation $\vec{\nu}_z$ coincides with the curl $\vec{w}$.

It can be seen from Fig. 1a that, in the flow of a microstructural liquid in a channel of sufficiently small cross section — for example, one for which $k < 5$ — the velocity profile may differ significantly from a Newtonian profile calculated for the same dp/dx and μ^n. This is because the energy spent on moving the liquid through the channel and characterizable by the quantity dp/dx is spent on overcoming friction due not only to linear shear, but also to the presence of moment stresses between particles. Thus, the particles of a microstructural liquid with the coefficients κ, μ, and γ have a lower linear velocity than microelements of a volume of Newtonian liquid located at the same points of the cross section, the Newtonian liquid here having a shear viscosity μ^n and flowing in the same channel under the influence only of a pressure gradient.

It follows from the foregoing that the viscous energy dissipation function of an MPL should be determined not only by the linear-velocity gradient but also by the microrotation characteristics. The expression for the viscous energy dissipation function of an incompressible MPL has the form [4]

$$\Phi^{mp}=t_{kl}(v_{l,k}-e_{klr}\nu_r)+m_{kl}\nu_{l,k}, \qquad (7)$$

where the stress tensor t_{kl} and micromoments tensor m_{kl} are determined as follows:

$$t_{kl}=\mu(v_{k,l}+v_{l,k})+\varkappa(v_{l,k}-e_{rkl}\nu_r),\quad m_{kl}=\beta\nu_{k,l}+\gamma\nu_{l,k}.$$

Here, the comma before an index denotes the operation of differentiation with respect to the corresponding tensor component.

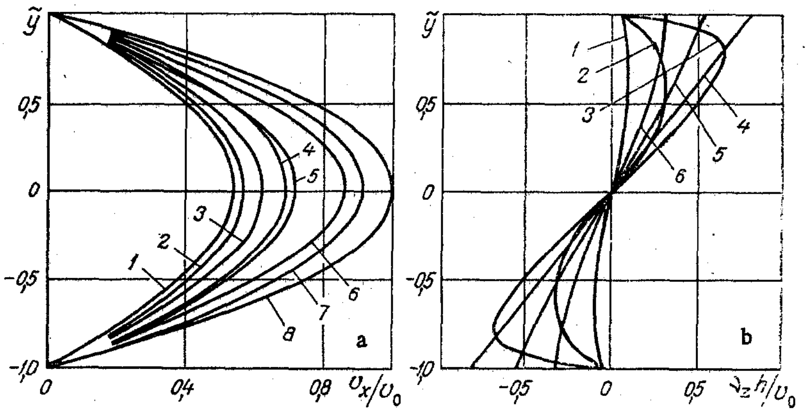

Fig. 1. Velocity v_x/v_0 (a) and microrotation $\nu_z h/v_0$ (b) with different values of δ_0, k, and ξ: a) k = 1, δ_0 = 2 and ξ = 1 (1), 0.7 (3), 0.5 (4), 0.1 (7); δ_0 = 2, ξ = 0.9, and k = 1 (2), 3 (5), 7 (6); curve 8 corresponds to a Newtonian liquid ($\delta_0 \to 0$). b) δ_0 = 2, ξ = 0.9, and k = 1 (1), 3 (2), 10 (3); k = 1, δ_0 = 2, and ξ = 0.1 (4), 0.3 (5), 0.5 (6).

Given the above flow conditions, Eq. (7) takes the form

$$\Phi^{mp}(y) = (\mu + \varkappa)\left(\frac{dv_x}{dy}\right)^2 + 2\varkappa\left(\frac{dv_x}{dy} + \nu_z\right)\nu_z + \gamma\left(\frac{d\nu_z}{dy}\right)^2. \tag{8}$$

Having replaced v_x and ν_z, after some simple transformations we obtain an expression for the viscous energy dissipation function of an MPL which characterizes the distribution of heat sources in the liquid volume:

$$\Phi^{mp}(\tilde{y}) = \frac{4v_0^2\mu^n}{h^2}\left\{\tilde{y}^2 - \frac{2\delta_0\xi}{(2+\delta_0\xi)\,\mathrm{sh}\,k}\left[\tilde{y}\,\mathrm{sh}\,k\tilde{y} + \frac{\mathrm{ch}\,k\tilde{y}}{k} - \frac{\xi}{2}\,\frac{2+\delta_0}{2+\delta_0\xi}\,\frac{\mathrm{ch}\,2k\tilde{y}}{\mathrm{sh}\,k} - \frac{2+\delta_0\xi}{2k^2\xi(2+\delta_0)}\,\mathrm{sh}\,k\right]\right\}. \tag{9}$$

Let one of the channel walls be of thickness H_1 and the other of thickness H_2. We assign a constant temperature T_c for the outside surfaces of both walls. We will state the problem of the heating of the liquid and the channel walls due to viscous energy dissipation in the region of stabilized heat exchange as follows:

$$\left.\begin{aligned} \lambda_q \frac{d^2T_2}{dy^2} &= -\Phi^{mp}_{(y)}, \\ \lambda_t \frac{d^2T_1}{dy^2} &= 0, \\ \lambda_t \frac{d^2T_3}{dy^2} &= 0. \end{aligned}\right\} \tag{10}$$

$$\left.\begin{aligned} &\lambda_q \frac{dT_2}{dy}\Big|_{y=h} = \lambda_t \frac{dT_1}{dy}\Big|_{y=h}, \quad \lambda_q \frac{dT_2}{dy}\Big|_{y=-h} = \lambda_t \frac{dT_3}{dy}\Big|_{y=-h}, \\ &T_1(h+H_1) = T_c, \; T_3(-h-H_2) = T_c, \\ &T_1(h) = T_2(h), \; T_2(-h) = T_3(-h). \end{aligned}\right\} \tag{11}$$

Here, the indices 1, 2, and 3 with T correspond to the temperature in different regions: in the plate of thickness H_1, in the liquid, and in the plate of thickness H_2, respectively.

Integrating system (10) and using boundary conditions (11), we obtain an expression for the temperature distribution in an MPL flowing in the channel:

$$\bar{T}_2(\tilde{y}) = W\left(\tilde{y}\,\mathrm{sh}\,k\tilde{y} - \frac{\mathrm{ch}\,k\tilde{y}}{k} - \frac{W}{16L}\,\mathrm{ch}\,2k\tilde{y}\right) + L\,\frac{1-\tilde{y}^2}{2} + \frac{1-\tilde{y}^4}{12} + C_1\tilde{y} + C_2, \tag{12}$$

where

$$W = \frac{2\delta_0\xi}{k^2(2+\delta_0\xi)\operatorname{sh}k};\ L = \frac{\delta_0}{k^2(2+\delta_0)};\ \bar{T}_2 = \frac{T_2 - T_c}{4v_0^2\mu^n}\lambda_q;$$

$$C_1 = M\frac{l_2 - l_1}{l_2 + l_1 + \frac{2\lambda_t}{\lambda_q}};\ C_2 = M - Q - (C_1 + M)\left(\frac{l_2\lambda_q}{\lambda_t} + 1\right);$$

$$M = kW\left(\operatorname{ch}k - \frac{W}{8L}\operatorname{sh}2k\right) - L - \frac{1}{3};$$

$$Q = W\left(\operatorname{sh}k - \frac{\operatorname{ch}k}{k} - \frac{W}{16L}\operatorname{ch}2k\right);\ l_1 = \frac{H_1}{h};\ l_2 = \frac{H_2}{h}.$$

Let us examine the special case $H_1 = H_2 = H$, when $l_1 = l_2 = l$ and Eq. (12) becomes

$$\tilde{T}_2(\tilde{y}) = W\left(W\frac{\operatorname{ch}2k - \operatorname{ch}2k\tilde{y}}{16L} + \frac{\operatorname{ch}k - \operatorname{ch}k\tilde{y}}{k} + \tilde{y}\operatorname{sh}k\tilde{y} - \operatorname{sh}k\right) + L\frac{1-\tilde{y}^2}{2} + \frac{1-\tilde{y}^4}{12} - \frac{Ml\lambda_q}{\lambda_t}. \quad (13)$$

Numerical analysis of (13) shows that when $\xi = 1$, i.e., in the case of "full adhesion" boundary conditions, the temperature in the channel with any δ_0 and k is always less than when it is calculated within the framework of the model of a Newtonian liquid:

$$\bar{T}_2^n(\tilde{y}) = \frac{1-\tilde{y}^4}{12} + \frac{l\lambda_q}{3\lambda_t}. \quad (14)$$

Here, a decrease in h is accompanied by an increase in $\bar{T}_2^n/\bar{T}_2$, and at the limit ($k = kh \to 0$)

$$\tilde{T}_2(\tilde{y}) \to \frac{2}{2+\delta_0}\bar{T}_2^n(\tilde{y}).$$

Obviously, for Poiseuille flows there should exist a range of physically permissible values of the boundary-conditions parameter α (or ξ) with assigned values of microstructural parameters δ_0 and k and dimension h. Thus, in cases of pronounced micropolarity (large δ_0 or small k and h), the amount of microrotation in the volume is substantially less than $\vec{w}$. This pertains also to microrotation of the liquid particles at the boundary, although the value here is a different percentage of the curl than in the volume of the liquid. As a result, α may not be close to unity in such cases.

Numerical analysis of (13) confirms, and Figs. 2 and 3 clearly illustrate, the above circumstance. As an example, we take $l = 100$ and a ratio of thermal conductivities $\lambda_t/\lambda_q = 78$, i.e., a ratio corresponding to steel and water. At the same time, we will henceforth use this value mainly without reference to any specific pair of liquid and channel material, taking it as a test value together with the other parameters δ_0, k, and ξ.

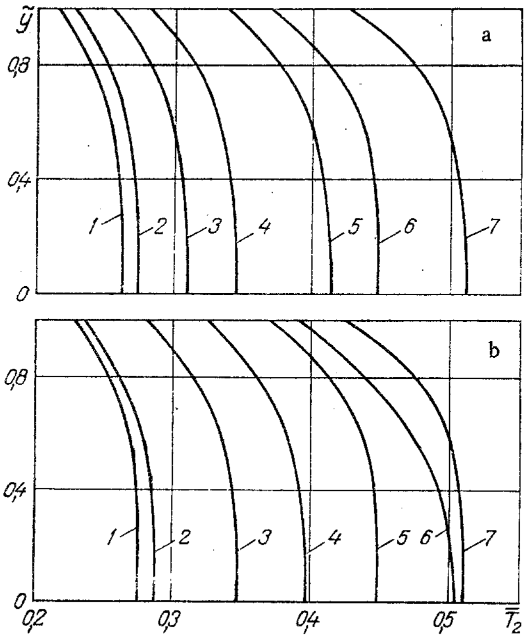

Fig. 2. Temperature in the channel: a) with $\delta_0 = 2$, $\xi = 1$, and k = 0.5 (1), 1 (2), 2 (3), 3 (4), 6 (5), 10 (6); b) with $\delta_0 = 2$, $\xi = 0.9$, and k = 1 (1), 0.3 (2), 3 (3), 5 (4), 10 (5), 0.1 (6); curves 7 correspond to a Newtonian liquid ($\delta_0 \to 0$).

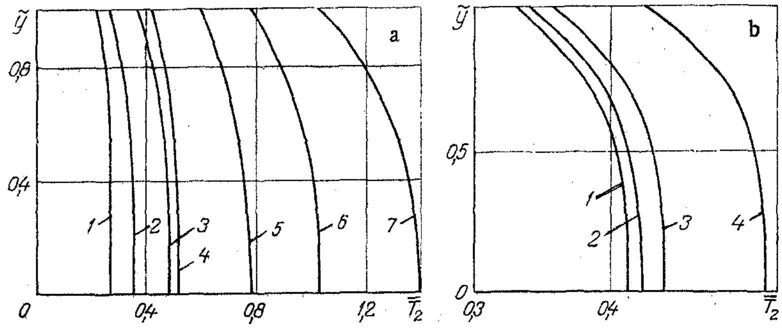

Fig. 3. Temperature in the channel: a) in an MPL with k = 1, $\delta_0 = 2$, and $\xi = 1$ (1), 0.6 (2), 0.4 (3), 0.2 (5), 0.1 (6), 10^{-5} (7); b) in water with k = 7, $\delta_0 = 14.5$, $\xi = 0.1$ (1), k = 7, $\delta_0 = 2.9$, $\xi = 0.5$ (2), and k = 7, $\delta_0 = 1.45$, $\xi = 1$ (3); curves 4 correspond to a Newtonian liquid ($\delta_0 \to 0$).

Figure 2a shows curves of the temperature field $\bar{T}_2(\tilde{y})$ for an MPL with the parameters $\delta_0 = 2$ and $k = \tilde{k}/h$ = const with $\alpha = 0$ flowing in channels of different dimensions h. Curve 7 corresponds to Eq. (14), i.e., to a calculation not allowing for the internal microstructure of the liquid. Any curve in Fig. 2a corresponds to a physically permissible combination of parameters δ_0, k, and ξ, which cannot be said of the curves in Fig. 2b, where $\xi < 1$. With a decrease in k and constant δ_0 and $\tilde{k}$, micropolarity is manifest to a greater degree by the liquid, and the microrotation of the particles in the liquid volume comprises a smaller and smaller part of $\vec{w}$. The value of $\vec{\vartheta}$ at the boundary should also account for a smaller part of $\vec{w}$. On the other hand, a formally prescribed value of α may correspond to a value of $\vec{\vartheta}$ which exceeds the fraction of $\vec{w}$ that is physically permissible for the given δ_0, k, and h. Thus, the character of the dependence of the curves of the temperature field $\bar{T}_2$ on k with constant ξ, δ_0, and $\tilde{k}$ in Fig. 2b is the same up to values of $k \simeq 1$ as in the case $\xi = 1$: the values of $\bar{T}_2$ decrease with a decrease in k. They then change, since when $k \lesssim 1$ micropolarity is strongly manifest in the liquid and the microrotation in both the liquid volume and at the boundary should be less than $[(1 - \xi)/2]\mathrm{rot}\,\vec{\vartheta}$. Thus, curves 2 and 6 correspond to physically unreal situations, and the flow of the given MPL in channels with cross-sectional dimensions determined as $h_1 = 0.3/\tilde{k}$ and $h_2 = 0.1/\tilde{k}$ should correspond to boundary-condition parameters $0 \leq \alpha < 0.1$.

Figure 3 illustrates the dependence of the temperature field on the boundary-conditions parameter. It is apparent (Fig. 3a) that $\xi \gtrsim 0.4$ when k = 1 and $\delta_0 = 2$. Thus, the boundary-conditions parameters will differ in the flow of an MPL in channels with substantially different cross-sectional dimensions. As a result, in Fig. 2, even in the case of physically permissible combinations of the parameters δ_0, k, and ξ, the representative curves can be regarded as corresponding to the flow of an MPL in different channels only within a certain limited range of k, when $\alpha \simeq$ const.

It was shown in [2] that $\tilde{k} = k/R_0 \simeq 7 \cdot 10^7$ m^{-1}, $\delta = 2\delta_0\xi/(2 + \delta_0\xi) \simeq 0.84$ for water flowing in a quartz microcapillary of radius $R_0 = 3 \cdot 10^{-7}$ m.

Thus, for water flowing in such a microcapillary, $\delta_0\xi \simeq 1.45$ and k = 7. Figure 3b shows curves of $\bar{T}_2(y)$ for water flowing in a steel capillary with $h = 10^{-7}$ m and a wall thickness 10^{-5} m. It should be noted that the smallness of the channel cross section (10^{-7}–10^{-6} m) at which the difference between $\bar{T}_2$ and $\bar{T}_2^n$ becomes substantial for water is due to the very small dimensions of its polymolecular structure formations, playing the role of "particles" of the MPL. These dimensions are obviously considerably larger for many other microstructural media (suspensions, liquid crystals, etc.).

In the above examples we used the value $\delta_0 = 2$. There are known to be [1] the following thermodynamic limitations on the material constants: $\kappa \geq 0$, $\mu^n \geq 0$. Thus, the range $0 < \delta_0 < \infty$ is theoretically possible. For example, with $l = 100$ and $\lambda_t/\lambda_q = 78$, we find that $\bar{T}_2^n(\tilde{y}) \gtrsim \bar{T}_2(y)$ by a factor of two in the case $\delta_0 = 4$, k = 1, and $\xi = 0.9$, and $\bar{T}_2^n(\tilde{y}) \gtrsim \bar{T}_2(y)$ by a factor of four in the case $\delta_0 = 20$, k = 1, and $\xi = 0.9$.

The decrease in the calculated dissipative heating of the microstructural liquid compared to the value obtained within the framework of the Newtonian liquid model is explained as follows. By determining the viscous energy dissipation function of the microstructural liquid and considering the latter to be Newtonian, we use overstated theoretical values of its flow velocity in microcapillaries. In actuality, the "equivalent" viscosity of such a liquid in microcapillaries is higher than handbook values of its shear viscosity [2]. As a result, despite the fact that Φ^{mp} is determined not only by the linear velocity but also by microrotation, their total contribution to Φ^{mp} is less than Φ^{n}, which is determined only by the linear-velocity gradient — since the theoretical values of the latter are higher than the actual values.

Consequently, allowing for the natural rotations of particles of a micropolar liquid leads to a substantial (with corresponding values of the microstructural parameters k and δ_0, as well as ξ) reduction in the theoretical values of its dissipative heating in the region of stabilized heat exchange.

NOTATION

2h, distance between plates; dp/dx, pressure gradient; v_x and ν_z, components of the velocity and microrotation vectors; μ, κ, β, and γ, material constants of the micropolar liquid; α, boundary-condition parameter; Φ, dissipative function; t_{kl}, stress tensor; m_{kl}, micromoments tensor; λ_t and λ_q, thermal conductivities of the materials of the channel and liquid; T, temperature; H_1 and H_2, thicknesses of channel walls; T_c, temperature of outside surfaces of channel; e_{rkl}, antisymmetric tensor; $\vec{w} = (1/2)\,\mathrm{rot}\,\vec{v}$, vorticity vector.

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THERMAL INTERACTION BETWEEN A PIPELINE AND THE SURROUNDING FROZEN GROUND

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A method is proposed for computing heat-transfer processes of pipelines and other engineering structures with finely dispersed frozen ground.

The exploitation of pipelines under low-temperature conditions of the surrounding ground is fraught with numerous complications. Reduction of the temperature of the product being transported can result in elevation of its viscosity (for oil), formation of ice (for water), and hydrated locks (for gases). Warming up the surrounding ground results in disturbance of its stability and, as a result, in pipeline buckling and undesirable ecological consequences. The most unfavorable are the pipeline exploitation conditions during its startup, when thermal losses are especially large. This same period is most complex from the viewpoint of the methodology of thermal design since nonstationary effects must be taken into account. These complexities grow significantly when the pipeline is in finely dispersed soils in which the phase transitions extend into the temperature spectrum.

Features of the thermal interaction between a pipeline and finely dispersed frozen soil are analyzed in this paper.

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